$\partial(\rho u)/\partial x = [\rho u]_x$; $\partial(\rho v)/\partial y = [\rho v]_y$;
$\partial u/\partial x = u_x$; $\bar{p} = (p - p_i)/(\rho_i u_i^2/2)$

Indices

w is the properties at walls:
e is the properties at outer boundary of boundary layer;
act is the properties obtained through the solution of the boundary-layer equation;
* is the properties calculated on the basis of the controlling temperature;
i is the properties of incoming flow;
in is the incompressible fluid.

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QUASISTEADY APPROACH IN CALCULATIONS FOR CONVECTIVE HEAT TRANSFER

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UDC 536.242

An equation is derived for determining the limits of applicability of the quasisteady approach in thermal calculations involving the cooling of metal plates in liquids and gases.

Under certain conditions of convective heat transfer, a boundary layer rapidly reacts to external perturbations and manages to change when there are changes in either the temperature of the object in the flow, the pressure at the inlet to the channel, or other parameters. In this case, "instantaneous steady states" exist at each time, and the steady-state approach can be used to determine the rate of the process. We call processes occurring under such conditions "quasisteady."

Whether a particular type of heat transfer can be treated as quasisteady is of both theoretical and practical interest. The use of equations found for the steady-state conditions substantially simplifies the calculations. There has been less study of unsteady heat-transfer processes, and dimensionless equations for the heat-transfer coefficients are not available for most unsteady processes.

The usual approach is to treat a heat-transfer process as quasisteady if the ratio of the Nusselt numbers found experimentally and calculated on the basis of the equations corresponding to the steady-state regimes is approximately unity [1-3]. In certain cases, the "condition for a quasisteady system" is assumed to be the approximate equality of the steady and unsteady heat fluxes [4, 5].

Attempts have been made to find the conditions under which the equations found for the steady-state conditions can be applied to unsteady heat-transfer processes. Comparing the heat fluxes calculated for the steady and unsteady regimes during the heating and cooling of a vertical plate, Sparrow and Gregg [4] found that the process can be treated as quasisteady under the condition

$$\frac{\dot{\Delta T}}{\Delta T}\left[\frac{xT_\infty}{g(\Delta T)}\right]^{1/2} \leqslant 0.033,$$

where

$$\Delta T = T - T_\infty;\quad \dot{\Delta T} = \frac{d(\Delta T)}{d\tau}.$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 5, pp. 857-860, November, 1976. Original article submitted October 14, 1975.

An analogous approach was taken in [6] in the case of heat transfer with forced convection. In this case the condition for the applicability of the equations of steady-state convection is

$$\frac{1}{6}\,\frac{\rho c\lambda t_0^2}{q_0^2}\left(\frac{t_0'}{t_0}+m\frac{v'}{v}\right)\to 0.$$

Here

$$t_0=T_0-T_1;\quad t_0'=\frac{dt_0}{d\tau};\quad v'=\frac{dv}{d\tau}.$$

The various aspects of the problem of unsteady heat transfer are studied in most detail in [3], where the experimental results were converted to the form

$$\frac{\mathrm{Nu}_u}{\mathrm{Nu}_{st}}=0{,}16\,K_{p*}^{0.24}$$

for comparison with the data of other investigators. Here

$$K_{p*}=K_p\frac{t_m}{t_q}\cdot 10^{-3},\quad K_p=\frac{\partial t_q}{\partial \tau}\cdot\frac{d^2}{(t_m-t_q)\,a_m}.$$

In the case $0.16\,K_{p*}^{0.24}\cong 1$ the process is quasisteady.

Interestingly, in most of the cases which have been treated it is necessary to know the time evolution of the temperature of the object in order to resolve the question of whether the equations found for the steady-state conditions can be used. However, the temperature of the object is generally unknown.

We believe that it is preferable to write the ratio $\mathrm{Nu_u}/\mathrm{Nu_{st}}$ as a function of the governing dimensionless numbers constructed from the properties substantially affecting the heat-transfer process but which are independent of the mechanism of this process. Among these parameters are the geometric and thermophysical properties of the system and the initial and boundary conditions.

As an example, we consider unsteady heat transfer during the cooling of a thin vertical plate in an unbounded medium. On the basis of physical considerations it is understandable that the ratio $\mathrm{Nu_u}/\mathrm{N_{st}}$ depends on the thermophysical properties of the plate and the medium at the initial time ($c_p, \lambda_p, \rho_p, c_m, \lambda_m, \rho_m, \beta, \nu$), the boundary and initial conditions ($t_{m\infty}$, t_{p0}), the geometric properties of the plate (δ and L), the acceleration due to gravity g, and the time τ.

We thus write

$$\frac{\mathrm{Nu}_u}{\mathrm{Nu}_{st}}=f(c_p,\lambda_p,\rho_p,c_m,\lambda_m,\rho_m,\beta,\nu,\delta,L,t_{m\infty},t_{p0},g,\tau).$$

Assuming a power-law functional relationship among these properties, and using the recommendations of theory of dimensionality, we find the following equation, making use of the analysis of [8]:

$$\frac{\mathrm{Nu}_u}{\mathrm{Nu}_{st}}=A\,(\mathrm{Gr}_0\mathrm{Pr}_0^{0.5})^n\left(\frac{\lambda_m}{\lambda_p}\,\frac{\delta}{L}\right)^p\mathrm{Fo}^r \tag{1}$$

The coefficient A and the exponents in Eq. (1) are to be determined experimentally. The plate is heated by briefly applying an electric current; then the plate cools because of free convection and radiation. The maximum plate temperature is 200°C, and the heating time is 0.4-1 sec. We study heat transfer of the plate in air, water, kerosene, and transformer oil. The plates are made of German silver and type 1Kh18N9T steel; the dimensions of the working part of the plate are $0.14\times 0.1\times 0.35\cdot 10^{-3}$ m.

The copper holders of the plate support also serve as current leads. The lower junction of the support is flexible to allow for elongation of the plate during heating.

The plate is heated by alternating current from a welding transformer (current of 800-1000 A; voltage of 42 V). Electrical and electromechanical timers are used to set and accurately measure the heating time. The plate temperature is measured by a copper–Constantan thermocouple (the diameter of the copper wire is 0.15 mm and that of the Constantan wire is 0.10 mm) and a single-point ÉPP-09M electronic potentiometer.

At the lower part of the plate is a horizontal isothermal shield, which shields against effects of the current leads. The position of the shield and the working thermocouple was chosen after preliminary experiments, control measurements of the temperature at various points along the height of the plate, and estimates of the influence of end effects through a calculation of a 2IGL-10 hydrointegrator.

For the experiments on the cooling of plates in liquids, we use a 60-liter tank of rectangular cross section fitted with windows.

The experiments were carried out by V. P. Pershin, M. A. Fadeev, and the author. The experimental results show that the Nusselt numbers for air are the same as those calculated from the equations for steady-state conditions, while those for water, oil, and kerosene are considerably lower. An analogous result was found by Sidorov [1]. In [3, 4] the ratio Nu_u/Nu_{st} turned out to be larger than one; it was shown in [5] that this ratio can be either smaller or larger than one, depending on the heat-transfer conditions and the direction of the heat flux.

The experimental results were treated by the method of least squares. The experimental data were approximated by a straight line, whose equation, calculated according to [7], is

$$K = 0.07 + 104\,N, \tag{2}$$

where

$$K = \frac{Nu_u}{Nu_{st}};\quad \frac{1}{N} = (Gr_0 Pr_0^{0.5})^{0.3}\left(Fo\,\frac{\lambda_m}{\lambda_p}\,\frac{\delta}{L}\right)^{0.15};\quad Fo = \frac{a_p\tau}{\delta^2}.$$

Equation (2) holds over the following ranges:

$$Gr_0 = 1.8\cdot 10^7 \div 1.0\cdot 10^9;\quad Pr_0 = 0.722 \div 200;\quad Fo = 5.0 \div 5.9\cdot 10^4;$$

$$\frac{\lambda_m}{\lambda_p} = 2.1\cdot 10^{-4} \div 3.8\cdot 10^{-2}.$$

For the experiments with water, the value of K lies in the range 0.15-0.20; for oil it lies in the range K = 0.25-0.5; and for kerosene it lies in the range K = 0.3-0.7. Only in the experiments on the cooling of plates in air do we find values of K approximately equal to one.

These results show that equations like (2) can be used; the use of these equations permits the engineer to determine whether a given process is quasisteady without carrying out special experiments. These results also confirm the hypothesis that the heating and cooling of objects in air can be assumed quasisteady over the temperature range occurring naturally.

NOTATION

x	is the coordinate (the x axis is along the plate);
T_∞	is the temperature of unperturbed medium;
T_0	is the temperature of main flow (beyond the boundary layer);
T_1	is the temperature of the surface in the flow, which is constant over time;
q_0	is the heat flux found from the steady-state equations;
v	is the velocity of main flow;
m	is the coefficient, equal to 1/2 for laminar flow and 1/5 for turbulent flow;
d	is the diameter;
c	is the specific heat;
λ	is the thermal conductivity;
ρ	is the density;
β	is the thermal-expansion coefficient;
ν	is the kinematic viscosity;
δ	is the plate thickness;
L	is the plate length;
g	is the acceleration due to gravity;
τ	is the time;
A, n, p, r	are the constants.

Indices:

p	are the properties of plate;
m	are the properties of medium.

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CALCULATION OF THE INTENSITY OF ELECTROMAGNETIC FIELDS OF THERMAL MICROWAVE DETECTORS AT HIGH TEMPERATURES. II

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UDC 621.372.8:536.21

The second moments are found for the spectral amplitudes of the thermal electromagnetic field of a dielectric inhomogeneity of complicated geometry heated to a temperature T.

The second moments of the spectral amplitudes of the thermal field of a dielectric structure with an arbitrary geometry consisting of steps and rods can be determined on the basis of the method of the generalized scattering matrix and [1, 2]. Specifying the temperature dependence $\varepsilon_j(T)$ of the dielectric constant of step j, we extend this method to the solution of analogous problems incorporating a temperature gradient in the inhomogeneities. Choosing as a basic inhomogeneity a dielectric inclusion of finite length, we can reduce the number of calculation procedures to a level about 2^n times lower than that for a semiinfinite step (here n is the number of elements in the structure selected).

1. Dielectric Inhomogeneity of Finite Length in a Waveguide

We seek a solution of the problem of the diffraction of an H_{p0} wave by a dielectric inclusion of bounded length in a rectangular waveguide (Fig. 1a) by the method of [1]. We make use of the symmetry of the inhomogeneity with respect to the plane z = d/2. We divide the incident field into parts of even and odd parity. This problem is reduced to two equivalent problems. The structure of the first problem is shown in Fig. 1b, where there is an electrical wall in the plane z = d/2. By placing a magnetic wall in the same plane, we find the geometry of the second problem. We denote by R^{-}_{mp} and R^{+}_{mp} the amplitudes of the harmonics of the waves reflected in region A, which are found through a solution of these two problems. According to the superposition principle, the amplitudes of the wave harmonics reflected from a dielectric inclusion of bounded length are

$$R_{mp} = (R^{+}_{mp} + R^{-}_{mp})/2,$$

and the amplitudes of the harmonics of the transmitted waves are

$$T_{mp} = (R^{+}_{mp} - R^{-}_{mp})/2.$$

Khar'kov State Scientific-Research Institute of Metrology. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 5, pp. 861-865, November, 1976. Original article submitted August 13, 1975.